

Assignment -1
Group- 3

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Ans-to-the-Q-No-1

* $L\left[\frac{3}{4}\right]$

Let,

$$f(t) = \frac{3}{4}$$

By definition of Laplace Transformation:

$$L[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

So,

$$L\left[\frac{3}{4}\right] = \int_0^{\infty} e^{-st} \left(\frac{3}{4}\right) dt$$

$$= \int_0^{\infty} e^{-st} dt$$

$$= \left[\frac{e^{-st}}{-s} \right]_0^{\infty}$$

$$= 0 - \left(\frac{3}{-4s} \right)$$

$$= \frac{3}{4s}$$

$$\text{Hence } L[1] = \frac{1}{s} ; s > 0$$

$$\therefore L\left[\frac{3}{4}\right] = \frac{3}{4s} ; s > 0$$

(Ans)

$$* \mathcal{L} \left[\frac{1}{2} \right]$$

Let,

$$f(t) = \frac{1}{2}$$

By definition of Laplace Transformation,

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

So,

$$\mathcal{L} \left[\frac{1}{2} \right] = \int_0^{\infty} e^{-st} \left(\frac{1}{2} \right) dt$$

$$= \int_0^{\infty} e^{-st} dt$$

$$= \left[\frac{e^{-st}}{-s} \right]_0^{\infty}$$

$$= 0 - \left(\frac{1}{-s} \right)$$

$$= \frac{1}{s}$$

Hence,

$$\mathcal{L}[1] = \frac{1}{s} ; s > 0$$

$$\mathcal{L} \left[\frac{1}{2} \right] = \frac{1}{2s} ; s > 0$$

(Ans)

Ans - to the - Q - No - 02

* $L[t^3]$

Let,

$$f(t) = t^3$$

By definition of Laplace Transformation,

$$L[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

So,

$$L[t^3] = \int_0^{\infty} e^{-st} t^3 dt$$

$$= \int_0^{\infty} e^{-x} \left(\frac{x}{s}\right)^3 \cdot \frac{dx}{s}$$

$$= \int_0^{\infty} e^{-x} \cdot \frac{x^3}{s^3} \cdot \frac{dx}{s}$$

$$= \int_0^{\infty} e^{-x} \cdot x^3 \cdot \frac{1}{s^{3+1}} dx$$

$$= \frac{1}{s^{3+1}} \int_0^{\infty} e^{-x} x^{(3+1)-1} dx$$

$$= \frac{\Gamma(3+1)}{s^{3+1}}$$

$$= \frac{\Gamma 4}{s^{3+1}}$$

Put,

$$st = x$$

$$\therefore t = \frac{x}{s}$$

$$\therefore dt = \frac{dx}{s}$$

As

$$t \rightarrow 0, x \rightarrow 0$$

$$t \rightarrow \infty, x \rightarrow \infty$$

By using Gamma function,

$$\Gamma^n = \int_0^{\infty} e^{-x} x^{n-1} dx$$

As

$$\Gamma(n+1) = n!$$

$$= \frac{3!}{s^{3+1}}$$

Hence,

$$L[t^3] = \frac{3!}{s^{3+1}}$$

$$= \frac{3 \cdot 2 \cdot 1}{s^4}$$

$$= \frac{6}{s^4}; s > 0$$

(Ans)

* $L[t^4]$

Let,

$$f(t) = t^4$$

By definition of Laplace transformation,

$$L[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

So,

$$L[t^4] = \int_0^{\infty} e^{-st} t^4 dt$$

$$= \int_0^{\infty} e^{-x} \left(\frac{x}{s}\right)^4 \cdot \frac{dx}{s}$$

put,

$$st = x$$

$$\therefore t = \frac{x}{s}$$

$$\therefore dt = \frac{dx}{s}$$

As

$$= \int_0^{\infty} e^{-x} \cdot \frac{x^4}{s^4} \cdot \frac{dx}{s}$$

$$= \int_0^{\infty} e^{-x} \cdot x^4 \cdot \frac{1}{s^{4+1}} \cdot dx$$

$$= \frac{1}{s^{4+1}} \int_0^{\infty} e^{-x} x^{(4+1)-1} dx$$

$$= \frac{\Gamma(4+1)}{s^{4+1}}$$

$$= \frac{\Gamma 5}{s^{4+1}}$$

$$= \frac{4!}{s^{4+1}}$$

Hence,

$$L[t^4] = \frac{4!}{s^{4+1}}$$

$$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{s^5}$$

$$= \frac{24}{s^5}; s > 0$$

(Ans)

$t \rightarrow 0, x \rightarrow 0$

$t \rightarrow \infty, x \rightarrow \infty$

By using Gamma function,

$$\Gamma^n = \int_0^n e^{-x} x^{n-1} dx$$

As

$$\Gamma(n+1) = n!$$

Ans-to-the-Q-No-3

$$* \mathcal{L}[e^{-3t}]$$

Let,

$$f(t) = e^{at}$$

By definition of Laplace transformation,

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

So,

$$\mathcal{L}[e^{-3t}] = \int_0^{\infty} e^{-st} \cdot e^{-3t} \cdot dt$$

$$= \int_0^{\infty} e^{-(s+3)t} dt$$

$$= \left[\frac{e^{-(s+3)t}}{-(s+3)} \right]_0^{\infty}$$

$$= 0 - \left[\frac{1}{-(s+3)} \right]$$

$$= \frac{1}{s+3} \quad ; \quad s > -3$$

(Ans)