

Assignment 01

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Laplace Transform

1. verify, $L\left[\frac{3}{4}\right]$

Pr solution: let, $f(t) = \frac{3}{4}$

By definition of Laplace Transformation

$$L[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

$$\text{So, } L\left[\frac{3}{4}\right] = \int_0^{\infty} e^{-st} \cdot \frac{3}{4} \cdot dt$$

$$= \frac{3}{4} \int_0^{\infty} e^{-st} dt$$

$$= \frac{3}{4} \left[\frac{e^{-st}}{-s} \right]_0^{\infty}$$

$$= \frac{3}{4} \cdot \frac{1}{-s} \left[e^{-st} \right]_0^{\infty}$$

$$= -\frac{3}{4s} (0 - 1)$$

$$= \frac{3}{4s} \quad \text{Ans:}$$

2 verify, $L\left[\frac{1}{2}\right]$

Solution: let, $f(t) = \frac{1}{2}$

By definition of Laplace Transform

$$L[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

$$L\left[\frac{1}{2}\right] = \int_0^{\infty} e^{-st} \cdot \frac{1}{2} \cdot dt$$

$$= \frac{1}{2} \int_0^{\infty} e^{-st} dt$$

$$= \frac{1}{2} \int_0^{\infty} \frac{e^{-st}}{-s} ds$$

$$= \frac{1}{2} \left[\frac{e^{-st}}{-s} \right]_0^{\infty}$$

$$= \frac{1}{2} \cdot \frac{1}{-s} \left[e^{-st} \right]_0^{\infty}$$

$$= \frac{1}{-2s} (0 - 1)$$

$$= \frac{1}{2s} \quad \text{Ans.}$$

3/ Verify, $L[t^3]$

Solution: Let. $f(t) = t^3$

By definition of Laplace Transform

$$L(t^n) = F(s) = \int_0^{\infty} e^{-st} f(t) \cdot dt$$

$$= \int_0^{\infty} e^{-st} t^3 dt$$

putting
 $st = x$
 $t = \frac{x}{s}$
 $\therefore dt = \frac{dx}{s}$

$$= \int_0^{\infty} e^{-st} \left(\frac{x}{s}\right)^3 \frac{dx}{s}$$

$$= \int_0^{\infty} e^{-x} \frac{x^3}{s^3} \frac{dx}{s}$$

$$= \int_0^{\infty} e^{-x} \cdot x^3 \cdot \frac{1}{s^3} \cdot \frac{1}{s} \cdot dx$$

$$= \frac{1}{s^4} \int_0^{\infty} e^{-x} x^3 dx$$

$$= \frac{1}{s^4} \Gamma_{n+1}$$

$$= \frac{1}{s^4} \cdot 3! = \frac{6}{s^4} \text{ Ans.}$$

using Gamma
 $\Gamma_{n+1} = \int_0^{\infty} e^{-x} x^n dx$
 $\therefore \Gamma_{n+1} = n!$

9. verify, $L[t^4]$

Solution: let $f(t) = t^4$

By definition of Laplace Transform

$$L[f(t)] = f(s) = \int_0^{\infty} e^{-st} f(t) dt$$

$$\text{So, } L[t^4] = \int_0^{\infty} e^{-st} \cdot t^4 dt \quad \left| \begin{array}{l} \text{putting,} \\ s1 = x \\ t = \frac{x}{s} \\ dt = \frac{dx}{s} \end{array} \right.$$

$$= \int_0^{\infty} e^{-x} \cdot \left(\frac{x}{s}\right)^4 \cdot \frac{dx}{s}$$

$$= \int_0^{\infty} e^{-x} \frac{x^4}{s^4} \frac{dx}{s}$$

$$= \int_0^{\infty} e^{-x} x^4 \cdot \frac{1}{s^4} \cdot \frac{1}{s} dx$$

$$\text{So, } L[t^4] = \frac{1}{s^5} \int_0^{\infty} e^{-x} x^4 dx \quad \left[\begin{array}{l} \text{By using} \\ \text{Gamma} \end{array} \right]$$

$$= \frac{1}{s^5} \Gamma(4+1)$$

$$= \frac{1}{s^5} 4! = \frac{24}{s^5}$$

5 verify, $L[e^{-3t}]$

Solution: let. $f(t) = e^{-3t}$

By de-finition Laplace Transform

$$L[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

$$\text{So, } L[e^{-3t}] = \int_0^{\infty} e^{-st} \cdot e^{-3t} dt$$

$$= \int_0^{\infty} e^{-st + (-3t)} dt$$

$$= \int_0^{\infty} e^{-st - 3t} dt$$

$$= \int_0^{\infty} e^{-(s+3)t} dt$$

$$= \left[\frac{e^{-(s+3)t}}{-(s+3)} \right]_0^{\infty}$$

$$\left[\int e^{mx} dx = \frac{e^{mx}}{m} \right]$$

$$= 0 - \left[\frac{1}{-(s+3)} \right] = \frac{1}{s+3} ; s > -3$$

Ans.